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RESEARCH PAPER

Estimating and Forecasting Meat Prices in Pakistan: A Comparative Study of ARIMA, GARCH and State Space ARIMA Models

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ABSTRACT

Forecasting plays essential role in making effective planning and decisions for a gainful business. Modeling monthly price series containing nonstationarity, seasonality, lag dependence, heteroscedasticity and structural changes is challenging. This leads to the applications of modeling and forecasting technique alternative to widely used conventional ARIMA (Autoregressive Integrated Moving Average) models and GARCH (generalized autoregressive conditional heteroscedastic) models to accommodate all these factors and capture the dynamics of the system. State space ARIMA models through Kalman Filter seem to be the appropriate candidates for this purpose. The main aim of this study is to investigate the worth of state space models in forecasting monthly meat prices in Pakistan under both homoscedasticity and heteroscedasticity. This study investigates a comparison of state space ARIMA model and ARIMA-GARCH models in the presence of heteroscedasticity and with simple ARIMA models after adjusting the heteroscedasticity through transformation. The empirical evidences are generated by applying these models to five monthly meat price series: chicken, mutton, beef, fish and shrimp in Pakistan. On the basis of the empirical results it is concluded that the state space ARIMA models outperform the ARIMA-GARCH models in the presence of conditional heteroscedasticity and simple ARIMA models in case of homoscedasticity.

Introduction

Pakistan is an agricultural country blessed with favorable environment and climatic conditions required for the growth of livestock. Live stock is considered as a central component of the agricultural sector of Pakistan. It is the major source of meat which in turn is considered as the prime source of protein, vitamins and certain minerals in non-vegetarian diet. The major sources of red meat are goat, buffalo, cattle and sheep in Pakistan. Due to high demand of red meat their production is continuously increasing. The estimated population of goat, buffalo, cattle and sheep reaches up to 78.2, 41.7, 44.4 and 30.1 million respectively in 2019-2020 from 74.1, 38.8, 46.1 and 30.5 million in 2017-18. In addition, beef and mutton production increased from 2155 and 717 thousand tons in 2017-18 to 2303 and 748 thousand tons in 2019-20. Poultry division is an essential and energetic section of agribusiness in Pakistan. It is the biggest and cheapest source of protein in Pakistan. This sector generates income and employment (direct /indirect) for about 1.5 million people. Poultry meat contributes 35 % of the total meat production with 9.1 % growth rate. Pakistan is the 11th largest poultry producer country in the world. Fishery plays a critical role in national economy of Pakistan, considered as deliberate income source of coastal inhabitants. In the Fiscal year 2019-20, the Fishing sector having a share of 2.06 % in agriculture value addition and 0.40 % in GDP, grew at 2.29 % but its exports add substantially to the national income (Pakistan Economic Survey, 2019-2020).

Pakistani meat has a high demand due to its exclusive taste. Pakistan has the ability to become international competitor in the red meat industry given the high demand for beef. Moreover, beef production is increasing significantly due to demand that is at the top in festive seasons. In addition, the worldwide outlook for halal meat is positive. Many halal products are now available in markets. Halal meat products have become a brand that is acceptable by non-muslims because of their positive association with health. Pakistan exports halal meat in almost all muslim countries therefore, the demand for mutton and beef meat are increasing day by day. Meat exports becomes a large source of earning foreign exchange. In financial years 2013-14, 2014-15, 2015-16, 2016-17 and 2017-18, Pakistan earned $211m, $230m, $243.m, $269m and $221 respectively from meat exports. (Mohiudin, 2018). Meat utilization in the year 2000 was 11.7 kg per capita in Pakistan that was increased by 13.8 kg in 2006 and 14.7 kg in 2009. It is expected that the utilization of meat will be reached at 40 kg per capita in 2020 (Sohaib and Jamil, 2017). The industrialization and capitalism have increased the pace of life in modern era. Masses have shifted to consumption of fast food rather than cooking at home. This has created an ever growing fast food industry which offers a huge market for meat suppliers. There are thousands of fast-food restaurants and chains profitably working in almost all cities of Pakistan. The international chains have been established in almost all major cities with ever increasing network; the include McDonalds, KFC, Hardees, Fat Burger, Burger King to name a few.

Literature Review

According to recent statistics, the fast-food industry has approximately 169 million consumers and is ranked 2nd largest in Pakistan and the 8th largest globally. It generates 16% of the total employment in the manufacturing sector. This industry with changing dynamics offers an even greater potential in the future economic growth (Ali, S, 2014). One of the important facts is that the fast-food industry is correlated with meat consumption. With the growth of fast food industry, the demand for meat products is significantly increased.

The discussion so far has made it clear that the meat industry has a central role in domestic economy as well as the exports. Therefore, meat price forecasting is very necessary for the government and investors to plan their activities in an effective manner. The fluctuations in the prices of meat are not only due to the demand, supply and consumption factors but also depend on many other unpredictable and irregular factors that are random in nature. Designing appropriate models for forecasting the meat prices is a challenging task due to stochastic and irregular form of meat prices. Forecasts based on an appropriate model provide the basis for effective planning and policy making. Conventional univariate autoregressive integrated moving average (ARIMA) models are widely used for forecasting a time series. The ARIMA models operate under the assumptions of homoscedasticity but mostly, the economic time series have non-constant variance. Mostly in economic time series, positive and negative news affect the variance differently which is known as the leverage effect. Autoregressive Conditional Heteroscedasticity (ARCH) models suggested by Engle (1982), generalized GARCH model by Bollerslev (1986) and its extensions are available in literature to model this phenomenon and providing effective forecasting ability (see Pasha, Qasim and Aslam, 2007 and references therein).

In economics and finance, a lot of problems are encountered in which the exact value of the variable of interest is unobservable. Furthermore, due to political issues and irregular events some structural changes in the underlying process occurred. In economics and finance, the lack of knowledge and forecasts concerning the exact value of the variable of interest and the relationship between variables leads to enormous difficulties. Furthermore, political issues and irregular events cause structural changes in the underlying process. In such a situation, static ARMA models cannot capture the changing characteristics of the system appropriately and some sort of dynamic models are required to accommodate these hidden changes. State space models (SSM) are one of these dynamic models. These models consisting of two equations: observed equation and state equation, describe the probabilistic interdependence between the observed measurement and the hidden state variable. The main function of state space models is to develop the state vector of the model that accommodates the hidden components of the system and summarizes it as a whole. In state space modeling, the chance of making specification error is minimum due to the fact that whether a variable affecting the system is observed or not, it automatically updates the system as the new observation comes. Since the state-space models depend on the whole history of the underlying data generating process, the estimation of its parameters is complicated. Kalman filter proposed by Kalman (1963) is an estimation algorithm for which the information about all the state history is not needed, it only depends upon the instant covariance matrix and instant previous state. The state space models through Kalman filter technique are a powerful source of forecasting a time series capturing its dynamic behaviour.

Conventionally, classical Box-Jenkins ARIMA time-series modeling technique is used to analyze the data collected over time in agriculture and in control engineering the state-space models have been used which accommodate the variations and unobservable factors. Several national and international researchers used different time series modeling and forecasting techniques in the field of agriculture, energy, environmental sciences and economics such as Mahmud and Qasim, 2009; Clement and Samuel, 2011; Miswan, Ngatiman, Hamzah and Zamzamin, 2014; Saini and Mittal, 2014; Nouman, 2014; Aamir and Shabri, 2015; Iqelan, 2015; Aamir and Shabri, 2016; Rehman, Jandong, Chandio, and Hussein, 2017. But to the best of our knowledge, state space have not so far been used to model time series data in Pakistani livestock fields. In this study, we have used GARCH-type models and state-space ARMA models to forecast meat prices in Pakistan to fulfill this gap. Our study has many folds: firstly we have applied the ARIMA modeling to the selected series. Secondly, we have applied the hybrid of ARIMA and GARCH models to accommodate the temporal dependence and heteroscedasticity simultaneously. Thirdly we have estimated the time varying variances using GARCH-type models and used these variances to transform the original heteroscedastic series into homoscedastic series. We have applied the state space ARIMA models to both homoscedastic and heteroscedastic series and compared the forecasting performance of these models with ARIMA- GARCH models in case of heteroscedasticity and ARIMA models under homoscedasticity to highlight the significance of these models.

Material and Methods

This study is related to the analysis of the monthly prices of chicken, fish, beef, shrimp and mutton based on per kilogram in Pakistan. The data are collected from the web www.indexmundi.com. It includes data regarding the price of chicken, shrimp and beef from Jan 1995 to May 2018, mutton and fish from January 1995 to March 2018 and June 2017 respectively. For chicken, shrimp and beef prices, data consist of 281 observations out of which 269 observations are used for estimation and remaining observations are used for forecasting. For the mutton and fish prices, data consist of 279 and 270 observations respectively out of which 267 and 258 observations are used for estimation and remaining observations are used for forecasting. The statistical software "Eviews7, Eviews10, Excel, and Gretl" are used for the purpose of the estimation, forecasting and graphical representation of the analysis.

One of the objectives of this study is to investigate the performance of SSM both in the presence of heteroscedasticity and homoscedasticity. The applied methodology is divided into two parts and has adopted two types of modeling approaches such as static models and state-space models. These models are applied to both heteroscedastic and homoscedastic data. In the context of static models, Box & Jenkins ARIMA models work under the assumption of homoscedasticity; but in time series it is commonly found that the variance of error term conditionally depends on the past variances and squared error terms. To solve the problem of heteroscedasticity, we have applied two approaches such as GARCH-type models and transformed method. For state space modeling SSARIMA models are applied.

Methodology related to ARIMA, GARCH-type models and transformed method are presented first, secondly the state space models and Kalman filter is discussed.

ARIMA Models

The Box & Jenkins ARMA models are applied when the series is stationary. The ARIMA model with (p, d, q) order is used when the time series is non-stationary and converted into stationary time series by integrating d times then the ARMA models are fitted and the resultant models are called autoregressive integrated moving average ARIMA(p, d, q). Let Y_t be stationary time series after integrating d times i.e. I(d) then ARMA(p, q) model to this series can be described as

$$Y_t = \varphi_o + \varphi_1 Y_{t-1} + \cdots + \varphi_p Y_{t-p} + \vartheta_1 \varepsilon_{t-1} + \cdots + \vartheta_1 \varepsilon_{t-q} + \varepsilon_t \tag{1}$$

The sample autocorrelation function (ACF) and partial autocorrelation functions (PACF) may be used for the selection of appropriate values of *p* and *q*.

GARCH-type Models

The generalized autoregressive conditional heteroscedastic (GARCH) models plays crucial role to model autoregressive conditional heteroscedasticity. Engle (1982) suggested the ARCH model to account for conditional heteroscedasticity. In these models the variance is expressed as the function of past squared innovations. Bollerslev (1986) proposed the extended form of the ARCH model that is known as GARCH model in which the variance of the innovation is not only the function of past square innovations but also the function of past variances. Let v_t^2 be the variance of the innovation ε_t in (1) then the GARCH (r, s) model, is defined as,

$$v_t^2 = \omega + \omega_1 \varepsilon_{t-1}^2 + \cdots + \omega_r \varepsilon_{t-r}^2 + \lambda_1 v_{t-1}^2 + \cdots + \lambda_s v_{t-s}^2 \tag{2}$$

where *r* and s represent the order of GARCH terms and the order of ARCH terms respectively.

In these models, the effect of positive shocks and negative shocks is equal on the variance hence called the symmetric GARCH models. However, in financial time series, negative news have large effects as compared to positive news on the variance, known as the leverage effect. Asymmetric GARCH models accommodate this fact. Three asymmetric GARCH models such as EGARCH, TARCH and PARCH are applied in this study to accommodate the leverage effect on the meat price series.

EGARCH Model

According to the study conducted by Nelson (1991), the EGARCH models constitute the first introduction of an asymmetric effect. The variance equation of this model is given as

$$ln v_t^2 = \omega + \sum_{i=1}^{r}(\omega_i|\eta_{t-i}|) + \sum_{j=1}^{s}\lambda_i ln v_{t-j}^2 \quad (3)$$

where $\eta_t = \frac{\varepsilon_t}{v_t}$ is the standardized residuals series.

TGARCH Model

The asymmetric TGARCH model proposed by Zakoian (1994) can be written in the form of variance equation as:

$$v_t^2 = \omega + \sum_{i=1}^{r}(\omega_i\varepsilon_{t-i}^2 + \gamma_i\varepsilon_{t-i}^2\Gamma_{t-i}) + \sum_{j=1}^{s}\lambda_j v_{t-j}^2 \quad (4)$$

where Γ_{t-k} is the indicator taking value zero when ε_t is positive and 1 when ε_t is negative.

PARCH model

The PARCH models proposed by Ding *et al.* (1993) is given as:

$$v_t^\delta = \omega + \sum_{i=1}^{p}\omega_i\,(|\varepsilon_{t-i}| - \gamma_i\varepsilon_{t-i})^\delta + \sum_{j=1}^{q}\lambda_j v_{t-j}^\delta \quad (5)$$

Transformation Method

The problem of heteroscedasticity can be removed by transforming the original series into a new series having constant variance. One way to remove heteroscedasticity is by using weights. If the pattern of the heteroscedasticity is known, the original series is divided by these weights and the new series generated in this method will be homoscedastic. Most often the pattern of the heteroscedasticity is not known especially in time series and have to estimate the weights and use these estimates for transformation explicitly. In our study, GARCH-type models are applied to estimate conditional variances. Let X_t be the original series and v_t^2 is estimated variance series then the transformed series is

$$W_t = \frac{X_t}{v_t}$$

The W_t should be homoscedastic. This method is preferable in case the estimation technique is run under the assumption of constant variance such as least square estimation technique, maximum likelihood estimation; as it may be difficult sometimes to apply GARCH model along with some nonlinear mean model.

State Space Models

A state space model provides a structure for examining a stochastic dynamical system that can be analyzed through a stochastic process. These models are defined by two equations: measurement equation and state equation

$$y_t = \boldsymbol{H}_t \, \boldsymbol{\beta}_t + \varsigma_t, \qquad \varsigma_t \sim N(0, \sigma_\varsigma^2)$$

$$\boldsymbol{\beta}_{t+1} = Z_t \, \boldsymbol{\beta}_t + \boldsymbol{d}_t + R_t \boldsymbol{\zeta}_t, \qquad \zeta_t \sim N(0, Q_t)$$

where y_t observation related to an m × 1 unobservable state vector $\boldsymbol{\beta}_t$. The matrix H_t is the state to observation linear transformation matrix, in the univariate case it becomes a row vector of order 1×m. It is assumed that ς_t and ζ_t are independent white noise processes, $\boldsymbol{\zeta}_t$ is the vector of white noise process with Q_t an m× m covariance matrix. The vector $\boldsymbol{d}_t$ is constant of size m × 1. The square matrix Z_t is the transition matrix of order m × m. The matrices $\boldsymbol{H}_t$, Z_t and $\boldsymbol{d}_t$, are time invariant in case of univariate time series (Box, Jenkins and Reinsel, 2011).

State Space ARMA Model

For the state space ARMA (*p*, *q*) model the system matrices and vectors in the above equations are as follows:

$$Z = \begin{bmatrix} \varphi_1 & \varphi_2 & \cdots & \varphi_p & \vartheta_1 & \vartheta_2 & \cdots & \vartheta_q \\ 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \qquad \beta_{t-1} = \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ \vdots \\ y_{t-p} \\ \zeta_{t-1} \\ \zeta_{t-2} \\ \vdots \\ \zeta_{t-q} \end{bmatrix}$$

$$\beta_t = \begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-p+1} \\ \zeta_t \\ \zeta_{t-1} \\ \vdots \\ \zeta_{t-q+1} \end{bmatrix} \qquad d = \begin{bmatrix} \varphi_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad R = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad H = [1 \quad 0 \; 0 \; 0 \; . \; . \; . \; 0]$$

The state equation of ARMA (p, q) is

$$\begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-p+1} \\ \zeta_t \\ \zeta_{t-1} \\ \vdots \\ \zeta_{t-q+1} \end{bmatrix} = \begin{bmatrix} \varphi_1 & \varphi_2 & \cdots & \varphi_p & \vartheta_1 & \vartheta_2 & \cdots & \vartheta_q \\ 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ \vdots \\ y_{t-p} \\ \zeta_{t-1} \\ \zeta_{t-2} \\ \vdots \\ \zeta_{t-q} \end{bmatrix} + \begin{bmatrix} \varphi_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \zeta_t$$

Kalman Filter

State space model depends upon whole history of the data so it cannot be estimated directly. To solve this problem, the powerful an iterative mathematical computational process known as the Kalman filter proposed by Kalman (1963) is available in literature to estimate the current state vector $\hat{\beta}_{t/t}$ and covariance matrix $P_{t|t}$. Starting from initial state estimator $\beta_0 = \hat{\beta}_{0|0}$ and covariance matrix P_0=$P_{0|0}$ the optimal $\hat{\beta}_{t/t}$ and $P_{t/t}$ are obtained by the following iterative procedure:

$$\hat{\beta}_{t/t} = \hat{\beta}_{t/t-1} + K_t(y_t - H_t\hat{\beta}_{t/t-1})$$

where

$$K_t = P_{t/t-1}H_t^{'}(H_tP_{t/t-1}H_t^{'} + \sigma_{\varsigma}^2)^{-1}$$

$$\hat{\beta}_{t/t-1} = Z_t\hat{\beta}_{t-1/t-1}$$

$$P_{t/t-1} = Z_tP_{t-1/t-1}Z_t^{'} + Q_t$$

and

$$P_{t/t}= P_{t/t-1} - P_{t/t-1}H_t^{'} (H_t P_{t/t-1}H_t^{'}+ \sigma_{\varsigma}^2)^{-1} H_tP_{t/t-1}$$

The matrix K_t is refered as the Kalman gian.

(for detail see Kalman, 1963).

Results and Discussion

The plots of all the series at level are given in Figure 1 showing the nonstationarity of the data. Figure 2 exhibits first log difference series showing stationary pattern for each series. For checking stationarityy of the data sets, the Augmented Dickey-Fuller(ADF) unit root test is also applied and the results are reported in Table 1. It is obvious that all the series are nonstationary at level and stationary at the first log difference.

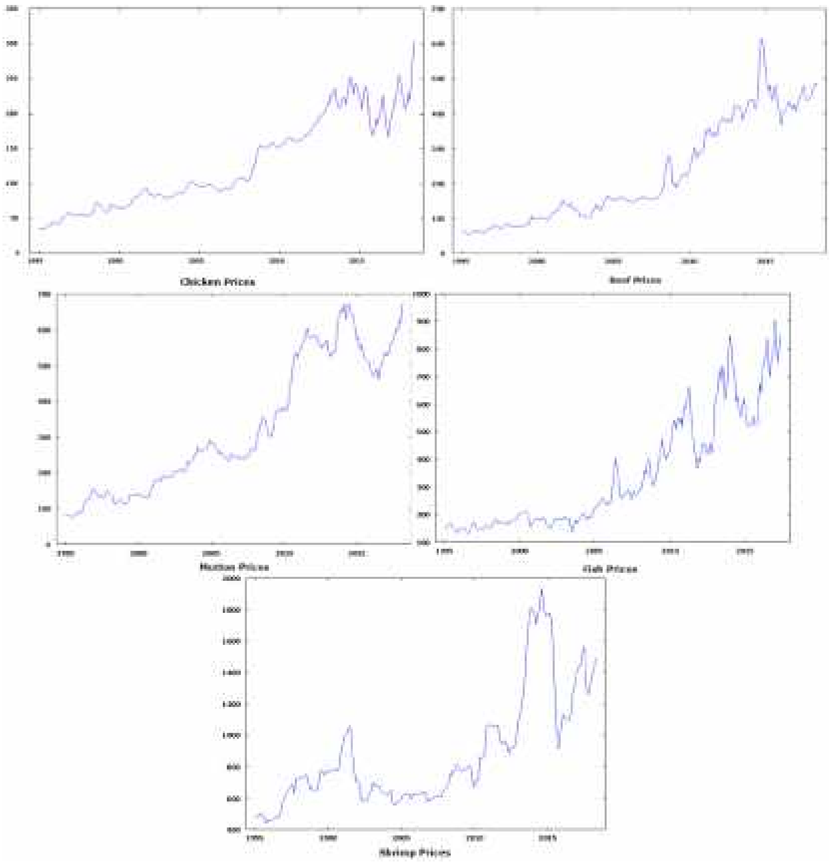
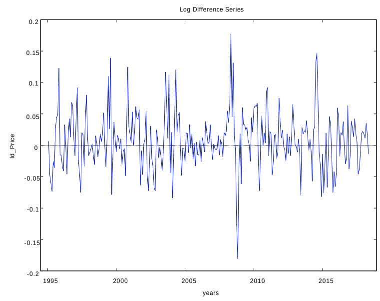
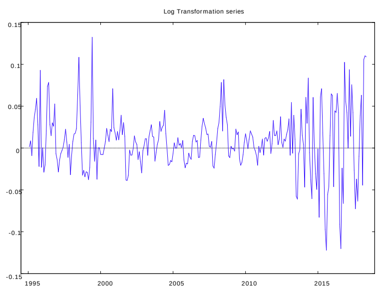


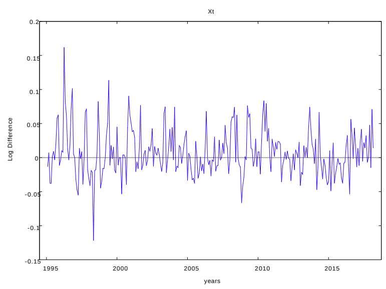
Figure 1: Monthly Meat Prices /Kg.



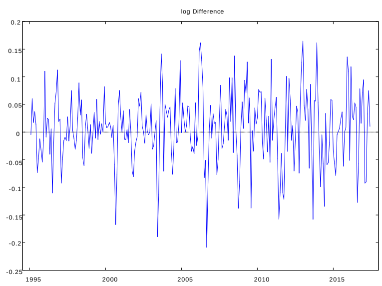
(A). Plot of beef Seri



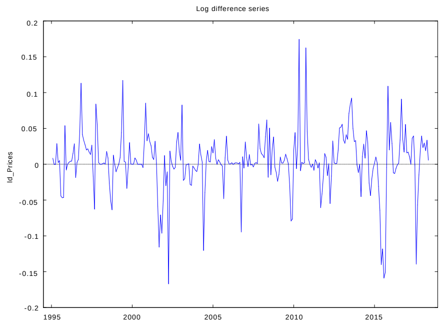
(B). Plot of Chicken Series



(C). Plot of Mutton Series.



(D). Plot of Fish Series



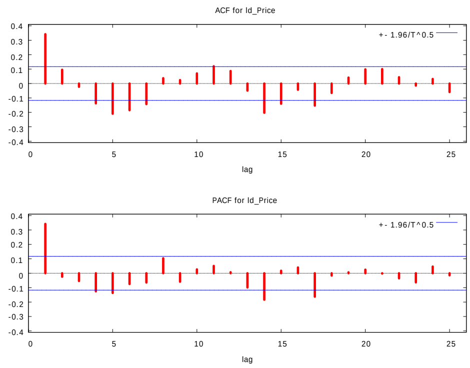
(E). Plot of Shrimp Series.

Figure 2: Plots of log difference Series of Meat Prices

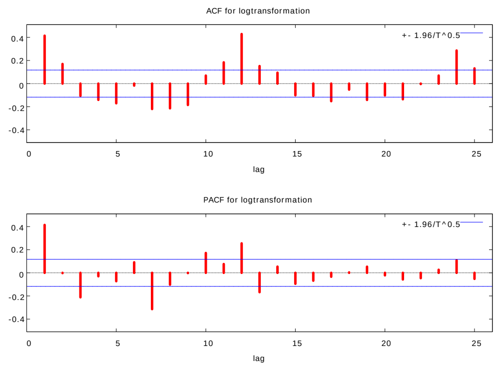
Table 1: Unit Root Test for All Meat Series.

Series	Before Log Difference		After Log Difference	
	t-Statistic	p-value	t-Statistic	p-value
Beef	-0.180138	0.9378	-11.65284	0.0000
Mutton	-0.003361	0.9566	-11.43019	0.0000
Chicken	0.270283	0.9764	-3.947347	0.0020
Fish	-0.593547	0.8685	-12.18159	0.0000
Shrimp	-1.705091	0.4276	-10.17128	0.0000

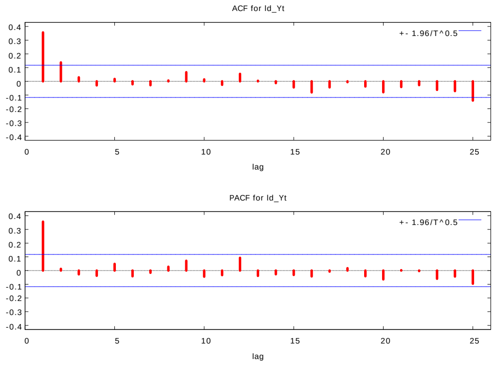
The correlogram of ACF and PACF show a seasonal pattern for the Beef, Chicken, and Fish series. Therefore, the seasonality component of the data set have been removed by using STL decomposition and TERMO/seat method. Figure 3 and Figure 4 illustrate the correlogram after and before the seasonal adjustment.



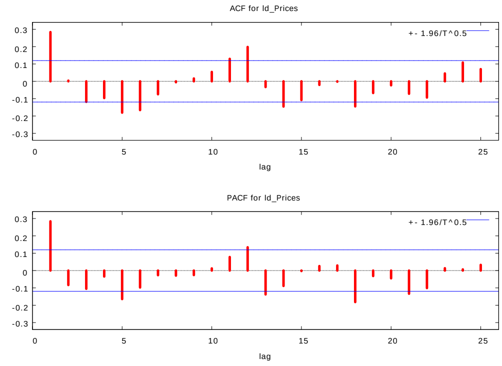
(A). Correlogram of beef series.



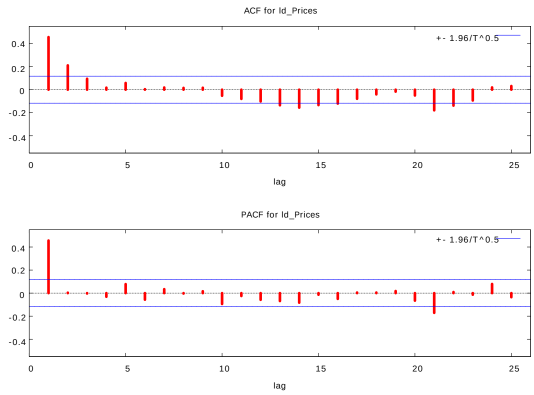
(B). Correlogram of chicken series



(C). Correlogram of Mutton series.

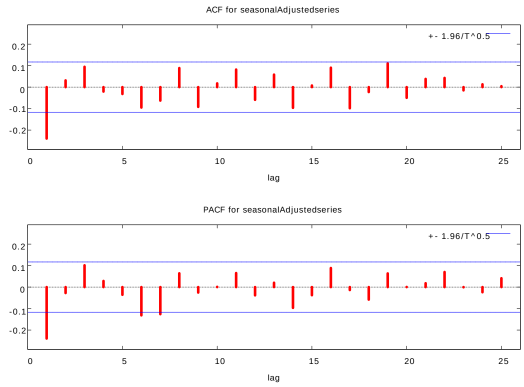


(D). Correlogram of Fish series.

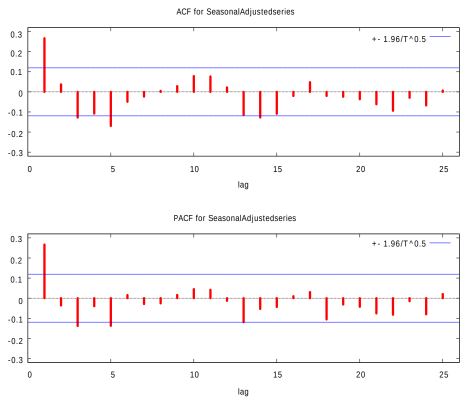


(E). Correlogram of Shrimp series.

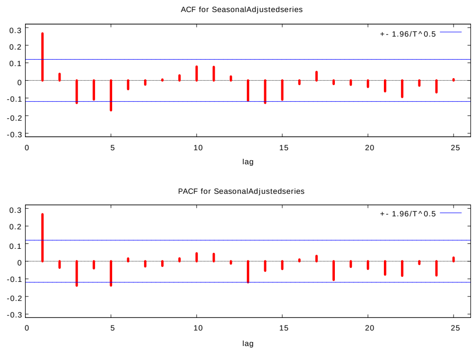
Figure 3: Correlogram of log Difference Series.



(A). Correlogram of beef series.



(B). Correlogram of chicken series



(c). Correlogram of Fish Series.

Figure 4: Correlogram of Seasonal Adjusted Series.

Table 2 reports the descriptive statistics for all the series. These results indicate that the distribution of each series is not normal for both at level and at first log difference. So quasi maximum likelihood estimation is used due to the fact that QMLE are asymptotically normal and efficient.

Table 2
Descriptive Statistics of Meat Price Series

Series/ at level	Mean	Median	Std.Dev.	Skewness	Kurtosis	Jarque-Bera	p-value
Chicken	127.1289	99.34000	64.20172	0.471252	1.945831	23.41181	0.000008
Beef	228.0210	158.1200	147.8604	0.627363	1.990679	30.36049	0.000000
Mutton	327.2572	262.6400	184.7054	0.403216	1.678817	27.85185	0.000001
Fish	347.1231	260.8800	207.9151	0.871896	2.519193	36.80982	0.000000
Shrimp	881.8559	754.6700	360.1399	1.2485	3.67844	78.3868	0.000000
After Achieving Stationarity							
Chicken	0.007297	0.004869	0.021920	0.902121	6.201918	158.1511	0.000000
Beef	0.007034	0.008130	0.045377	0.217409	5.010769	49.37633	0.000000
Mutton	0.007446	0.001206	0.034263	0.669865	4.708662	54.60849	0.000000
Fish	0.006556	0.007266	0.057709	-0.31002	3.995871	15.42510	0.000447
Shrimp	0.0040	0.0025	0.0419	-0.5948	7.5616	259.2772	0.00000

After achieving the stationarity, removing the seasonality of the series. We have applied ARMA *(p, q)* models with a suitable order and observed the correlogram of ACF and PACF of the residual and squared residuals of these models. The selected best ARMA model for each series is given in Table 3. For the residuals of these models, the diagnostics criteria are fulfilled except the value of ACF of the squared residuals is significant as exhibited in Figure 5(a) and 5(b). Only for the mutton series, the selected model has fulfilled all the diagnostics. These results indicate the existence of conditional heteroscedasticity in all the series except mutton series.

Table 3
Selected ARMA Models for All Meat Series

Series	Model	AIC
Beef	ARMA(4,4)	-4.614223
Chicken	ARMA(2,3)	-5.036373
Mutton	ARMA(1,0)	-4.024998

Fish	ARMA(3,5)	-3.009096
Shrimp	ARMA (1,0)	-3.736590

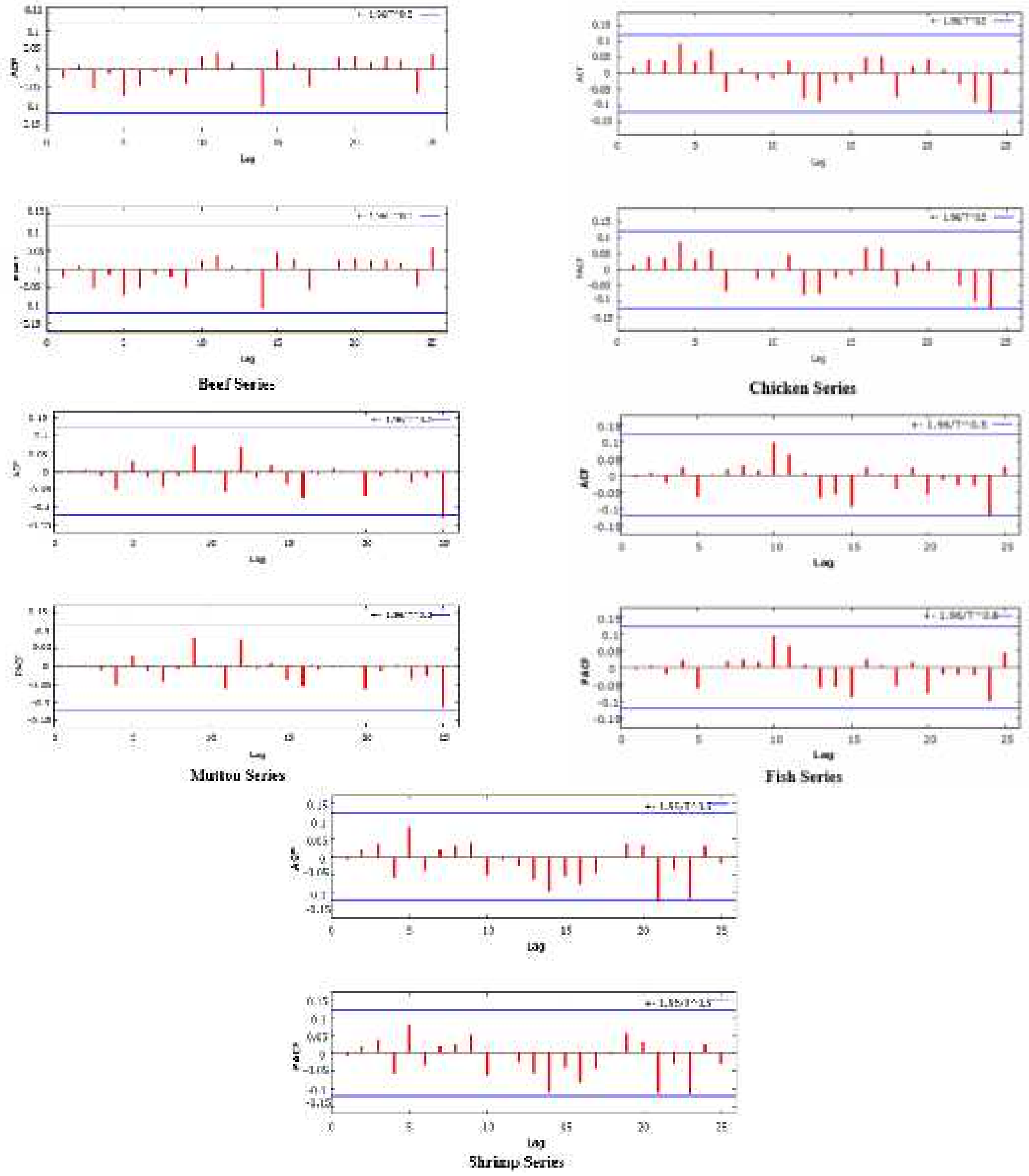


Figure 5(a): Correlogram of Residuals for ARMA models

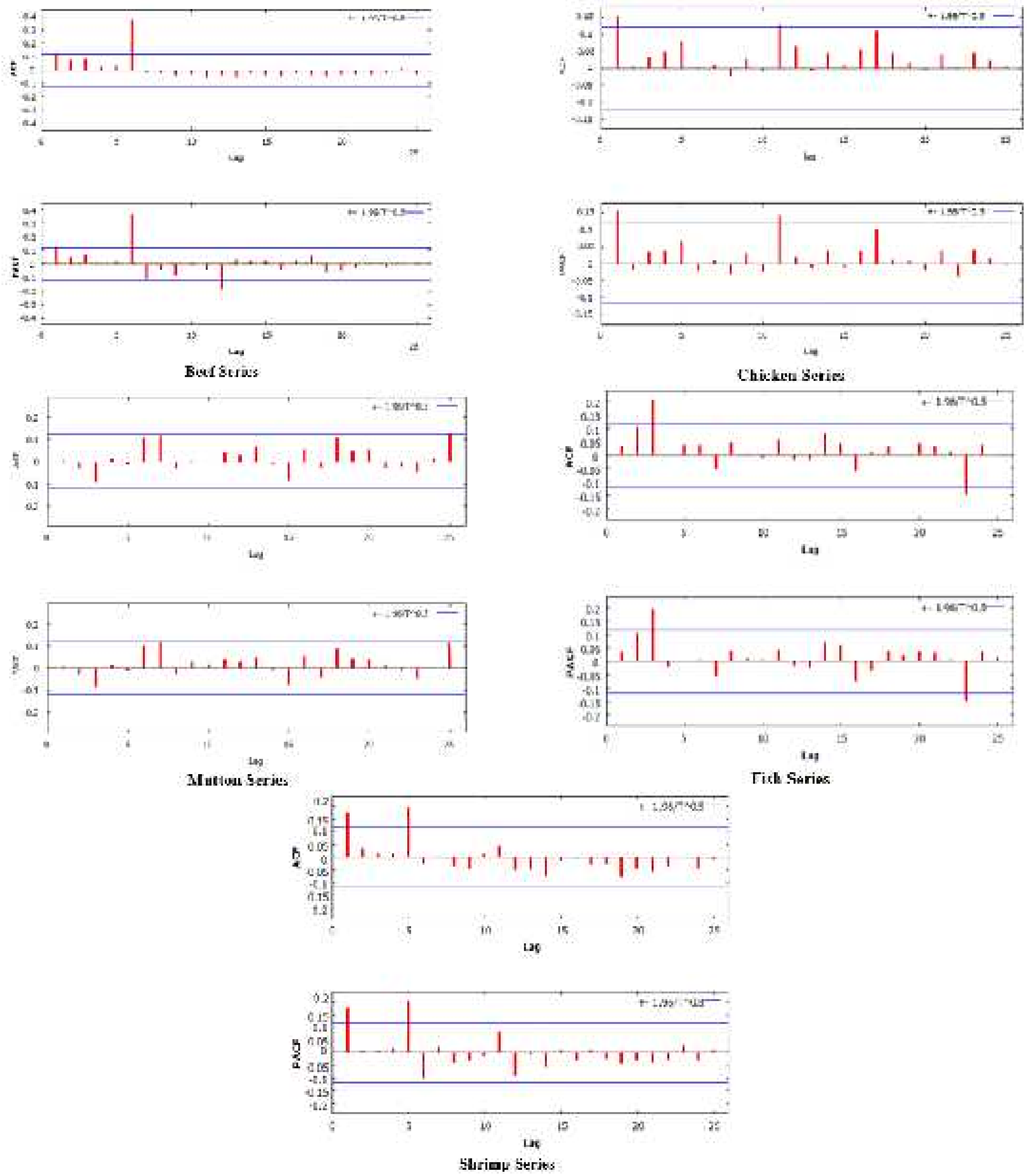


Figure 5(b): Correlogram of Squared Residuals for ARMA models

The significance of the ACF of squared residuals exhibit the conditional heteroscedasticity. The GARCH models are applied to model this heteroscedasticity to all the series except mutton series. The ARMA models with different orders are applied along with symmetric GARCH models with different orders and the diagnostics have been checked, the values of ACF and PACF of the residuals are insignificant for all the models. But for the beef and fish series the values of ACF and PACF of squared residuals are significant at some lags. To overcome this problem the ARMA models with different orders are applied with asymmetric GARCH models, EGARCH, TGARCH and PARCH with different orders. The selected models for each series are presented in Table 4. These selected model fulfilled residual diagnostic criteria which is shown in Figure 6(a) and 6(b). These results demonstrate that asymmetric effect in the variance is substantial for the beef and fish series.

Table 4
Selected Models for GARCH-type of All Meat Series

Series	Model	AIC
Beef	ARMA (3,5)-PARCH (1,1,1)	-4.925391
Chicken	ARMA (3,2)-GARCH (1,1)	-5.306631
Mutton	-	-
Fish	ARMA (3,5)-TARCH (1,1,2)	-3.019394
Shrimp	ARMA (4,2)-GARCH (1,1)	-3.815045

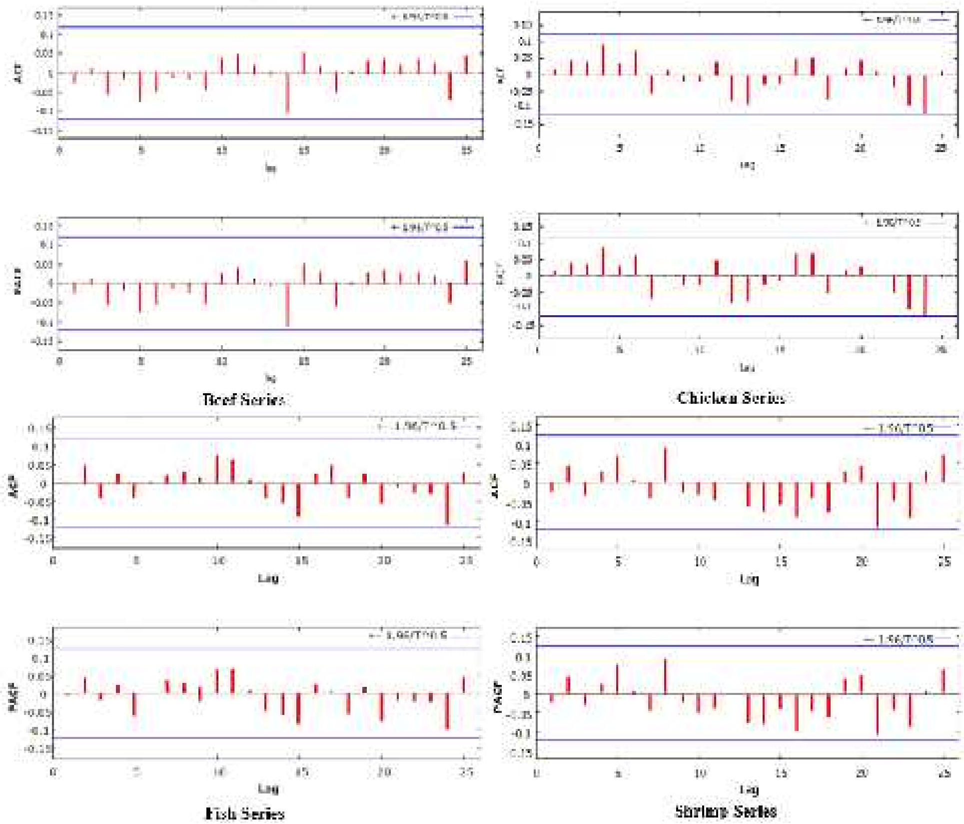


Figure 6(a): Correlogram of Residual for GARCH-type models

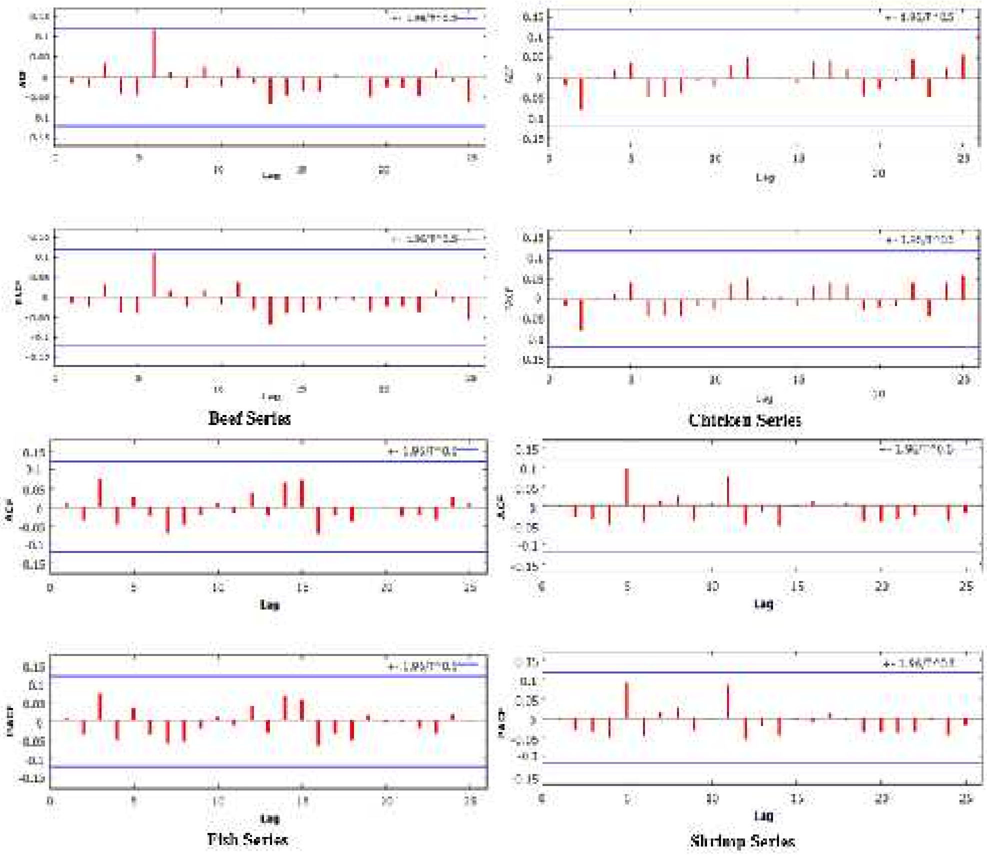


Figure 6(b): Correlogram of Squared Residuals for GARCH-type models

Transformation and ARMA Modeling

For the purpose of removing heteroscedasticity, the standard deviations series is estimated using selected ARMA-GARCH-type models in each case. The transformed series W_t is generated by dividing the stationary and depersonalized series by these estimated series. When applied the ARMA model to the weighted series the resulted model is called WARMA model. We have estimated a large number of tentative WARMA models and select a best model on the basis of AIC criteria for each series. The selected WARMA models are displayed in Table 5. The residuals of these models are independent and no more heteroscedasticity is left in meat prices series which can be seen in Figure 7 (a) and 7(b).

Table 5
Selected WARMA Models for Meat Series

Series	Model	AIC
Beef	WARMA(4,4)	2.846390
Chicken	WARMA(4,2)	2.929733
Mutton	-	-
Fish	WARMA(1,3)	2.921737
Shrimp	WARMA (5,4)	2.802051

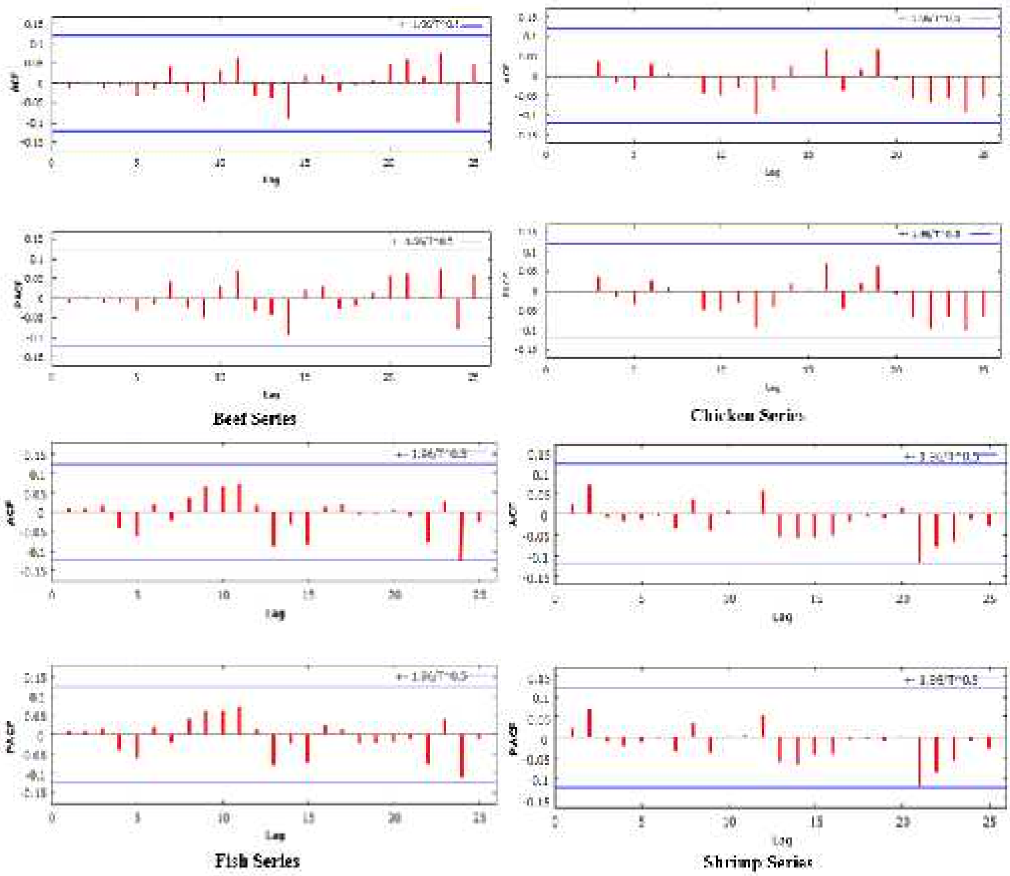
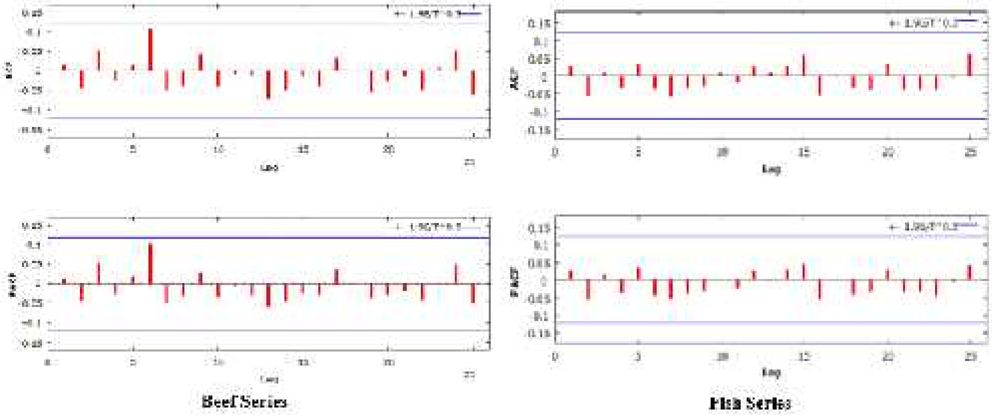


Figure 7(a): Correlogram of Residual for WARMA models



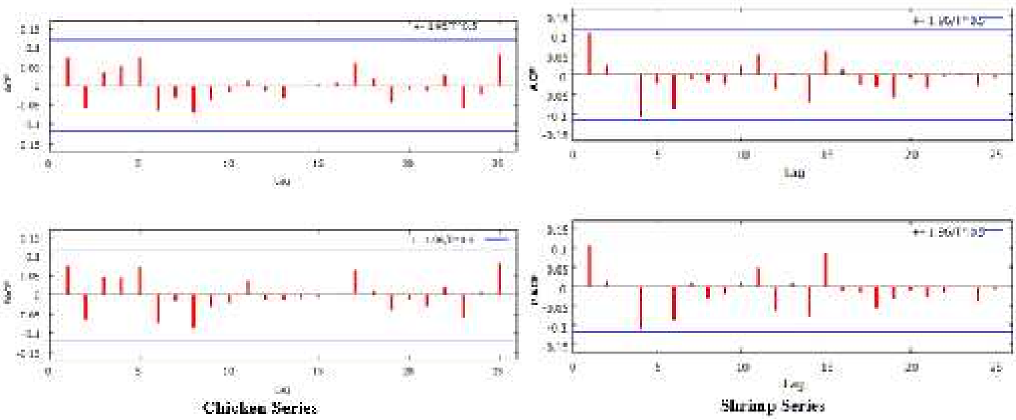


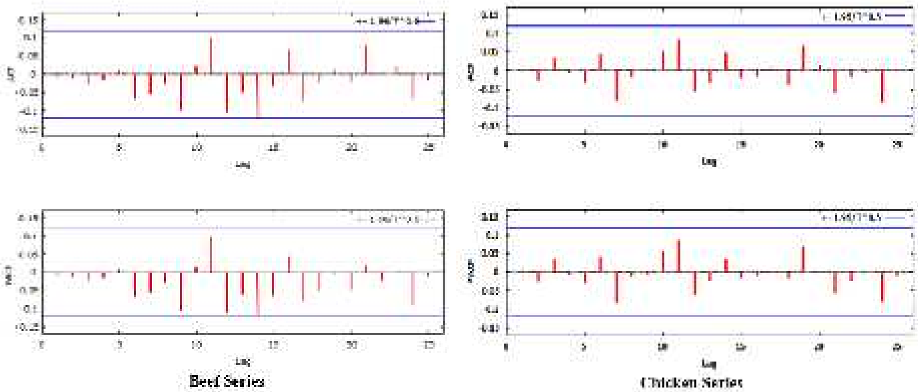
Figure 7(b): Correlogram of Squared Residual for WARMA Models

State Space Modeling

The State space ARMA models are applied to all the stationary and seasonally adjusted series and called the resultant models as SSARMA models. A large number of tentative SSARMA with different orders are estimated. The best model is selected on the basis of minimum values of AIC in each series and reported in Table 5. The correlogram of predicted residual and predicted squared residuals are observed and shown in Figure 8(a) and 8(b) which demonstrate the independency and homoscedasticity of the residuals of selected models.

Table 6
Final State Space ARMA Models for All Meat Series

Series	Model	AIC
Beef	SSARMA (4,5)	-4.53876
Chicken	SSARMA (5,5)	-5.86657
Mutton	SSARMA(1,0)	-4.01564
Fish	SSARMA (1,3)	-3.06857
Shrimp	SSARMA(1,0)	-3.73993



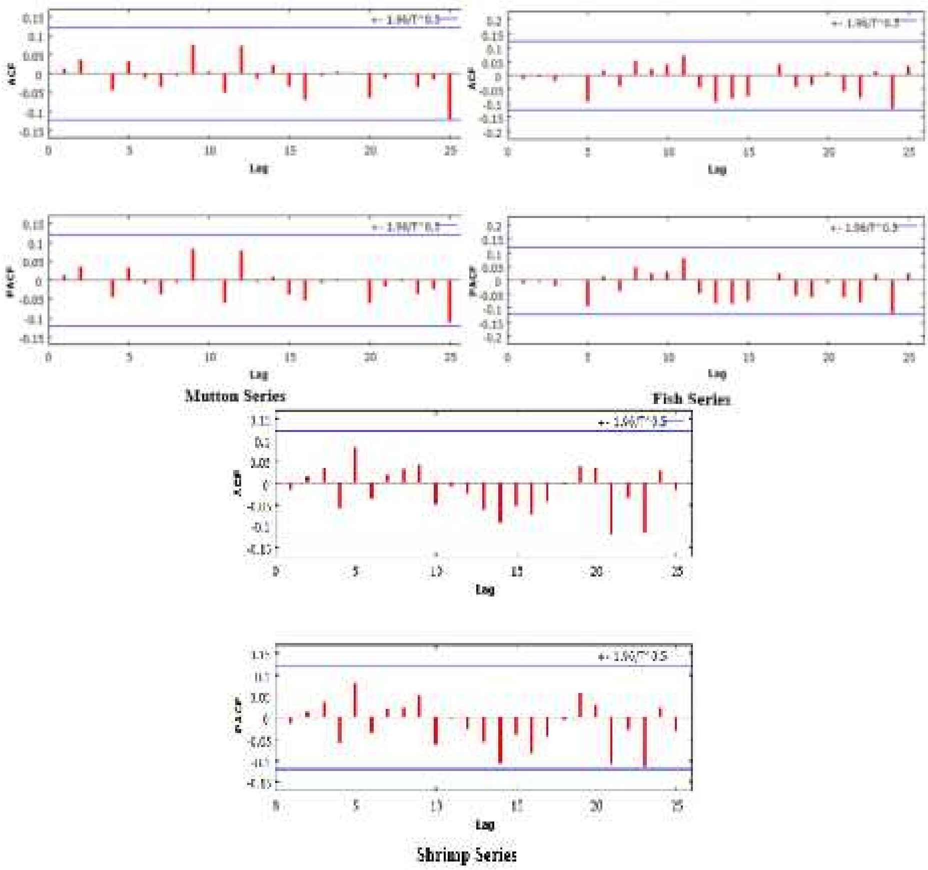
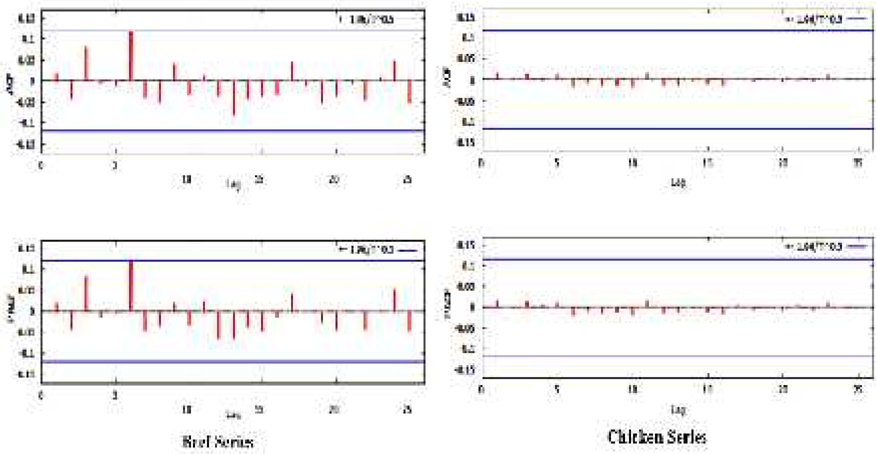


Figure 8(a): Correlogram of Residual for SSARMA Models



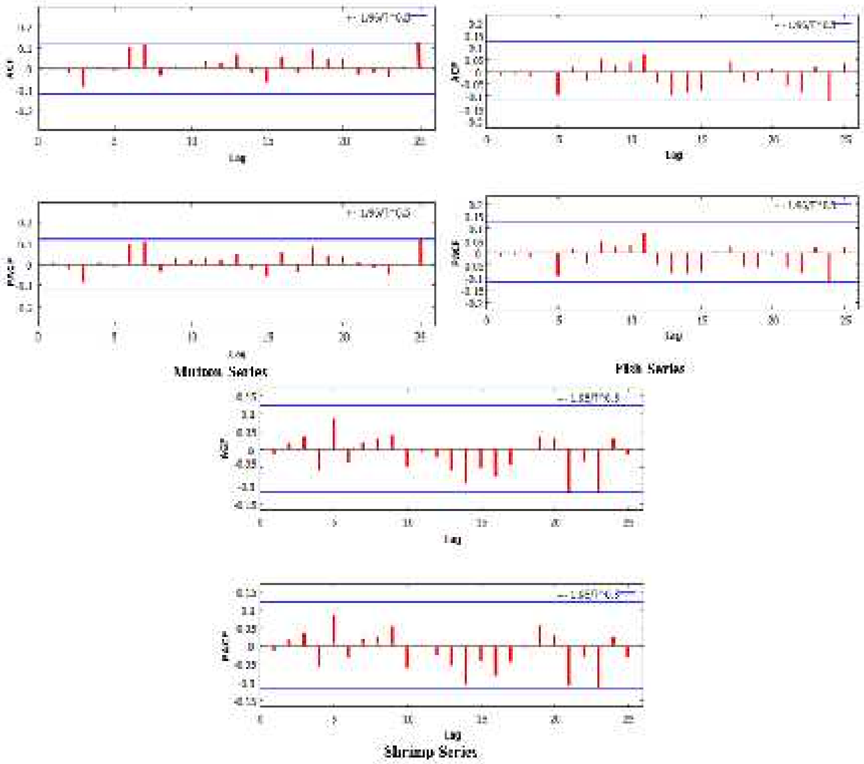


Figure 8(b): Correlogram of Squared Residual for SSARMA Models

The SSARMA models are also applied to the weighted series W_t. The resultant models are called SSWARMA models. A large number of tentative SSWARMA with different orders are estimated. The best model fore each series on the basis of AIC is reported in Table 6. In Figures 9(a) and 9(b), the correlograms of predicted residual and predicted squared residuals for SSWARMA models are presented which show that the residuals for all these models are independent and homoscedastic.

Table 6
Final State Space WARMA Models for Meat Series

Series	Model	AIC
Beef	SSWARMA(1,5)	2.878723
Chicken	SSWARMA(2,5)	2.983940
Mutton	SSWARMA(1,0)	-4.015649
Fish	SSWARMA(2,5)	2.936957
Shrimp	SSWARMA(1,2)	2.810596

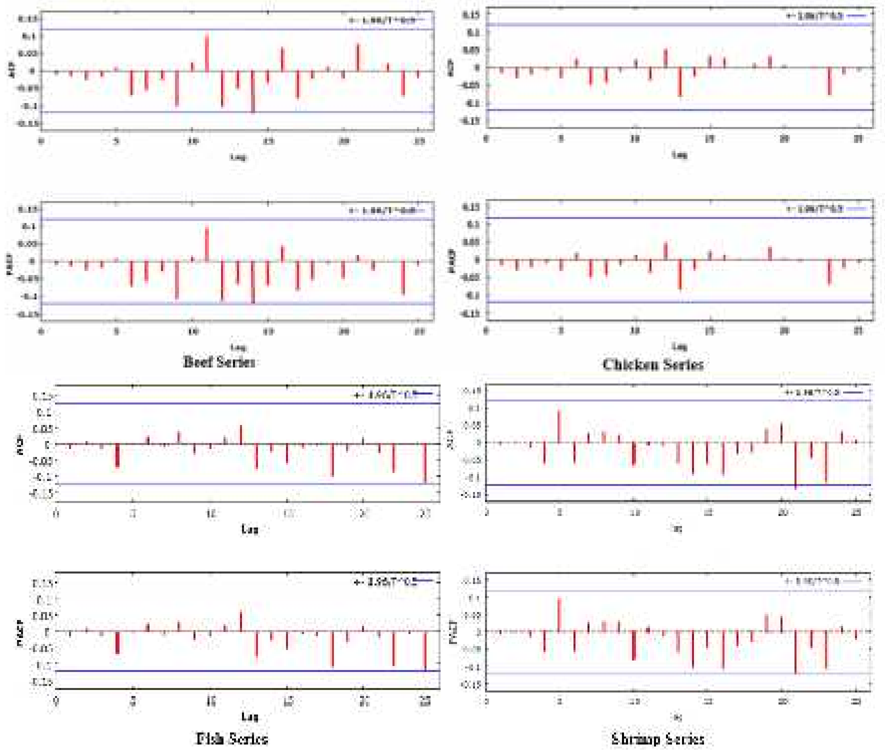


Figure 9(a): Correlogram of Residual for SSWARMA Models

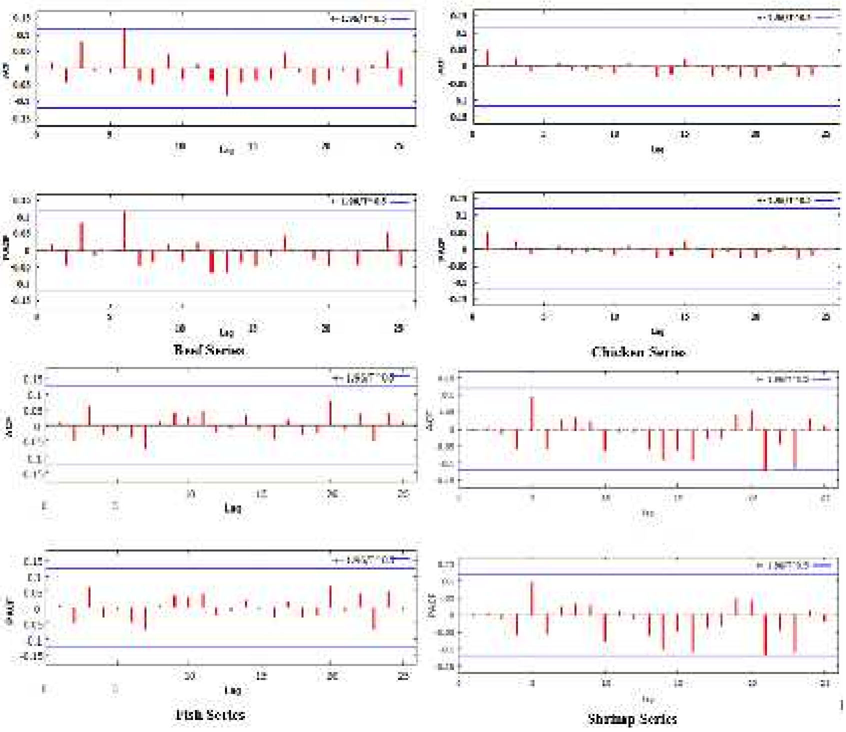


Figure 9(b): Correlogram of Squared Residual for SSWARMA Models

Forecasts Evaluation

The forecasting evaluation results based on root mean square error(Root.MSE), mean absolute error (MA. Error) and mean absolute percentage error (MAP. Error) of the selected models, both in case of static and state space context under heteroscedasticity are reported in Table 7. The lowest value in each case indicates the best model in terms of forecasting ability. The best model is selected as best on the basis of majority of the loss functions. For all the series the SSARMA models better performed to forecast the corresponding series than the ARMA models along with GARCH-type models. Table 8 represents the results for the forecast evaluation in case of transformed series considered in this study. In different series, different GARCH-type models are selected and taken as weights to transform the data series. It is obvious that in all the cases state space models performs better than the static models.

Table 7
Forecast Evaluation for Heteroscedastic Series

Series	Forecast Sample	Model	Root. MSE	MA. Error	MAP. Error
Beef	2017m06 2018m05	ARMA(3,5)-PARCH(1,1,1)	0.035031	0.019848	211.2778
		SSARMA(4,5)	0.033554	0.020004	173.1199
Chicken	2017m06 2018m05	ARMA(3,2)-GARCH(1,1)	0.023150	0.017970	111.3746
		SSARMA(5,5)	0.011079	0.008701	235.0948
Mutton	2017m04 2018m03	ARMA (1,0)	0.031761	0.026188	210.8256
		SSARMA(1,0)	0.031837	0.024913	101.3594
Fish	2016m07 2017m06	ARMA(3,5)-TARCH(1,1,2)	0.086932	0.074258	148.0740
		SSARMA(1,3)	0.060663	0.049682	136.2113
Shrimp	2017m06 2018m05	ARMA(4,2)-GARCH(1,1)	0.050094	0.033586	160.4759
		SSARMA(1,0)	0.04957	0.03476	133.3723

Table 8
Forecast Evaluation for Transformed Series (Homoscedastic)

Series	Forecast Sample	Selected Model	Root. MSE	MA. Error	MAP. Error
Beef	2017m06 to 2018m05	WARMA(4,4)	1.520059	0.994810	227.8097
		SSWARMA(1,5)	1.509543	0.874275	229.5290

Chicken	2017m06 to 2018m05	WARMA(4,2)	0.986019	0.909440	172.4031
		SSWARMA(2,5)	0.904227	0.821728	255.8328
Fish	2016m07 to 2017m06	WARMA(1,3)	1.196123	0.972756	110.7050
		SSWARMA(2,5)	1.145526	0.935874	100.7726
Shrimp	2017m06 to 2018m05	WARMA(4,5)	1.494543	1.035940	244.8314
		SSWARMA(1,2)	1.323744	0.871519	134.4996

Conclusion

The main goal of this study is to investigate the performance of state space ARMA models in the context of both heteroscedasticity and homoscedasticity. To achieve our goal, we have compared the performance of ARMA models both in the context of classical and state space for modeling and forecasting the meat price index in Pakistan. We have taken prices series of five different types of meat which are mostly used in Pakistan such as beef, chicken, mutton, fish and shrimps price series. Empirical results demonstrated that all the series except mutton price series are subject to conditional heteroscedasticity. To solve this issue two approaches are applied: firstly, the conditional heteroscedasticity is modeled along with ARMA models using symmetrical and asymmetrical GARCH models. In the case of asymmetric GARCH models, EGARCH, TGARCH, and PARCH models are applied. Secondly in each case the respective series of standard deviations is generated on the basis of selected ARMA model with GARCH-type model and the original series have been transformed into homoscedastic series and the ARMA models are applied to each homoscedastic series. It is worthwhile to note that the GARCH- type models are considered the powerful tools to model the conditional heteroscedasticity in literature, in this study it is also empirically proved that these models also provide the source of removal of heteroscedasticity which is the essential requirement in many estimation techniques.

To assess the forecasting performance of state space ARMA models a comparison is made between SSARMA models and ARMA with GARCH type models for heteroscedastic series and between SSARMA and ARMA models for homoscedastic series. It is worthy to note that the state space models not only successfully resolve the problem of heteroscedasticity in the sense that even the results for ordinary SSARMA models shows that there left no heteroscedasticity in the residuals of the fitted models but also outperform the ARMA models with GARCH-type models which are considered the best models to fit the conditional heteroscedasticity. Thus on the basis of empirical results derived in this study, it is concluded that:

- The SSARMA models have both qualities of ARMA models and state space models and provide the best ability to forecast the meat prices in Pakistan.
- The GARCH-type models are mostly used models in literature to handle the conditional heteroscedasticity existing in time series data. However, in the

present study simple state space models also control the conditional heteroscedasticity and outperform the classical models with GARCH errors both in the symmetric and asymmetric case showing that state space models have the quality to accommodate not only the conditional heteroscedasticity but also the leverage effect in the variance. This investigation is a big contribution in literature and lead to promote these models for other fields of life for modeling purpose.

- Moreover, in case of all the homoscedastic series, SSARMA models give approximately more efficient results in forecasting meat price index as compare to static models providing the best forecasting ability under homoscedasticity.
- Despite the worth fullness of the state space models, unfortunately there is lack of research work in this field. Possessing the properties of capturing dynamic of the system by updating the changings due to sudden jumps, political and economic fluctuations and intervention the use of state space models should be promoted in the fields of economics, finance, actuarial sciences, physical sciences, and biological sciences.

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